- Write two possible mechanisms for the radioactive decay of ^{55}Fe to ^{55}Mn.

Marks 5

Electron capture: $^{55}_{26}Fe + ^{0}_{-1}e \rightarrow ^{55}_{25}Mn$

Positron emission: $^{55}_{26}Fe \rightarrow ^{55}_{25}Mn + ^{0}_{+1}e$

The activity of an isotopically pure 1.000 g sample of ^{55}Fe is measured as 8.750×10^{13} Bq. Calculate the half-life (in days) of ^{55}Fe.
(The molar mass of ^{55}Fe is 54.94 g mol^{-1}.)

As the atomic mass of isotopically pure ^{55}Fe is 54.94 g mol^{-1}, the number of moles in 1.000 g is:

$$\text{number of moles} = \frac{\text{mass (g)}}{\text{atomic mass (g mol}^{-1})} = \frac{1.000\text{ g}}{54.94\text{ g mol}^{-1}} = 0.01820\text{ g mol}^{-1}$$

This corresponds to:

$$\begin{aligned}\text{number of nuclei} &= \text{number of moles} \times \text{Avogadro's number} \\ &= (0.01820\text{ mol}) \times (6.022 \times 10^{23}\text{ mol}^{-1}) = 1.096 \times 10^{22}\text{ nuclei}\end{aligned}$$

The activity (A) is related to the number of nuclei (N) by $A = \lambda N$ where λ is the decay constant. Hence,

$$\lambda = \frac{A}{N} = \frac{8.750\times10^{13}\text{ Bq}}{1.096\times10^{22}\text{ nuclei}} = 7.983 \times 10^{-9}\text{ s}^{-1}$$

The half life, $t_{½}$, is related to the decay constant by $t_{½} = \ln2/\lambda$. Hence,

$$t_{½} = \ln2/(7.983 \times 10^{-9}) = 8.683 \times 10^{7}\text{ s}$$

As 1 day is (24 × 60 × 60) s, this corresponds to

$$t_{½} = \frac{8.683 \times 10^{7}\text{ s}}{(24\times60\times60)\text{ s day}^{-1}} = 1005\text{ days}$$

Answer: **1005 days**

ANSWER CONTINUES ON THE NEXT PAGE

How many years will it take for the activity of this pure 1.000 g sample of ^{55}Fe to drop to 1.000×10^{9} Bq?

The number of radioactive nuclei decays with time according to $\ln(N_0/N_t) = \lambda t$. As the activity is proportional to the number of nuclei ($A = \lambda N$), this can be rewritten as:

$$\ln(A_0/A_t) = \lambda t$$

As $\lambda = 7.983 \times 10^{-9}$ s^{-1}, the activity will decay from $A_0 = 8.750 \times 10^{13}$ Bq to $A_t = 1.000 \times 10^{9}$ Bq in a time t where

$$\ln(8.750 \times 10^{13}/1.000 \times 10^{9}) = (7.983 \times 10^{-9}\ \mathrm{s}^{-1})t$$

$$t = 1.425 \times 10^{9}\ \mathrm{s}$$

As 1 year is (365 × 24 × 60 × 60) s, this corresponds to:

$$t = 1.425 \times 10^{9} / (365 \times 24 \times 60 \times 60) = 45.19 \text{ years}$$

Answer: **45.19 years**