**Marks**
**4**

- Moseley discovered experimentally in 1913 that the atomic number, $Z$, of an element is inversely proportional to the square root of the wavelength, λ, of fluorescent X-rays emitted when an electron drops from the $n$ = 2 to the $n$ = 1 shell.

$$i.e. \quad \frac{1}{\sqrt{\lambda}} = kZ$$

Derive an expression for the constant of proportionality, $k$, for a hydrogen-like atom which would allow the value of $k$ to be theoretically calculated.

**Squaring Moseley's relationship gives** $\frac{1}{\lambda} = (kZ)^2$ **(1)**

**The energy of an X-ray with wavelength** $\lambda$ **is given by** $E = \frac{hc}{\lambda}$**. Substituting in Moseley's value for** $\frac{1}{\lambda}$ **from (1) gives:**

$$E = hc(kZ)^2 \quad (2)$$

**For a hydrogen like atom, an electron in an orbital with quantum number** $n$ **has energy** $E = -Z^2E_R\left(\frac{1}{n^2}\right)$ **where** $E_R$ **is the Rydberg constant.**

**The energy *emitted* when an electron moves from an orbital with quantum number** $n$ **= 2 to an orbital with quantum number** $n$ **= 1 is:**

$$E = [-Z^2E_R\left(\frac{1}{2^2}\right)] - [-Z^2E_R\left(\frac{1}{1^2}\right)] = Z^2E_R\left(\frac{3}{4}\right) \quad (3)$$

**Equating equations (2) and (3) gives:**

$$hc(kZ)^2 = Z^2E_R\left(\frac{3}{4}\right)$$

**Rearranging for** $k$ **gives:**

$$k = \sqrt{\frac{3E_R}{4hc}}$$