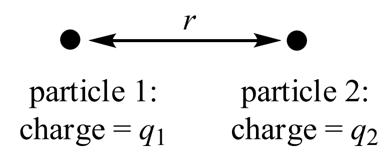
CHEM1901/3 Worksheet 3: The Energy Levels Of Electrons

Model 1: Two charged Particles Separated by a Distance *r*

According to Coulomb, the potential energy of two stationary particles with charges q_1 and q_2 separated by a distance r is:



$$V = k \times \frac{q_1 q_2}{r}$$

where k is a positive-valued proportionality constant [$k = (4\pi\varepsilon_0)^{-1}$].

Critical thinking questions

1. What happens to the magnitude of V if r is increased?

2. What is the value of V if the particles are separated by an infinite distance (i.e. $r = \infty$)?

3. Is $V < 0$ or is $V > 0$ if the particles have the *same* charge?

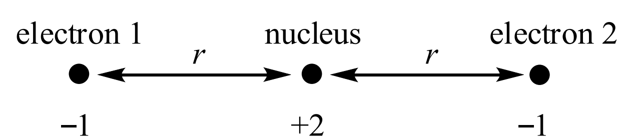
4. If $q = -e$ for an electron, what is q for a proton?

5. A hydrogen atom consists of an electron orbiting around a proton. Is the potential energy of a hydrogen atom positive or negative?

6. Using your answers to questions 2 and 5, describe in *words* what happens to the potential energy of a hydrogen atom as its electron is removed (i.e. the atom is ionized)?

7. The picture opposite shows two electrons and a helium nucleus arranged in a straight line.

   Write down the total potential energy of this arrangement as a sum of *three* terms and simplify your expression.



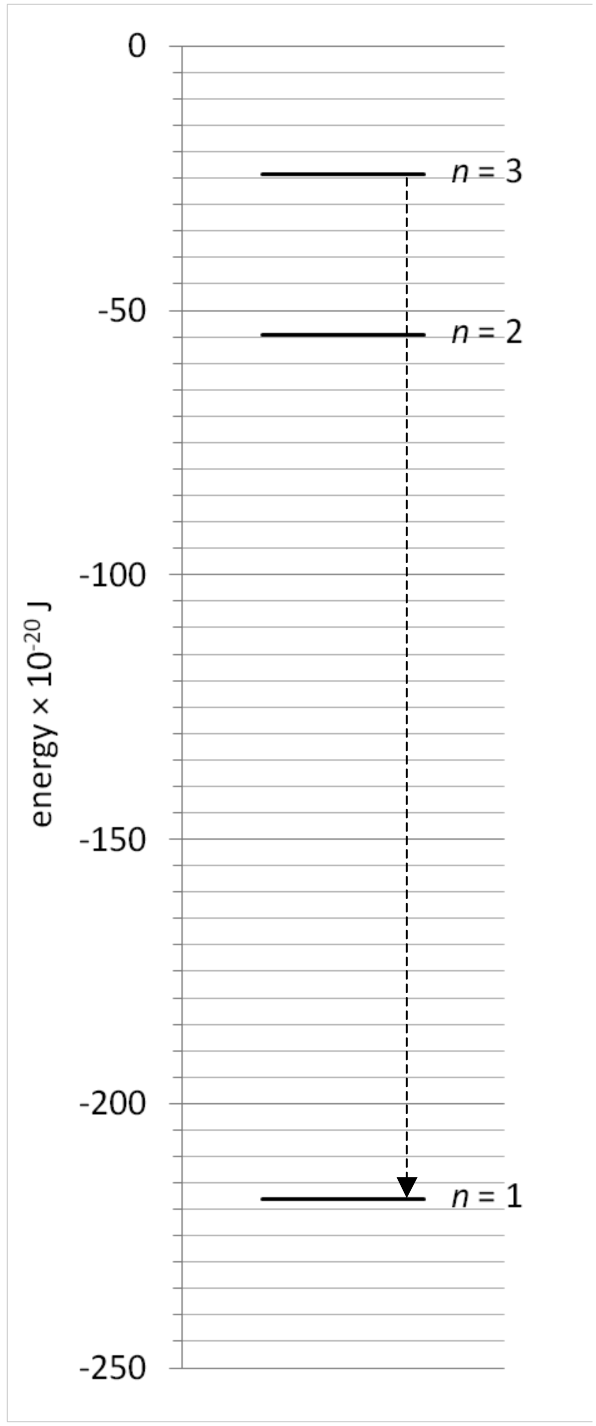
Model 2: Electron Energy

For an atom, such as hydrogen, with one electron orbiting around a nucleus with charge Z, the energy of the electron is given by the equation below:

$$E_n = -(2.18 \times 10^{-18}\ \text{J}) \frac{Z^2}{n^2}$$

where n = 1, 2, 3, 4.... The different values of n correspond to the *allowed* energies that the electron can have. These energies are called "energy levels". The lowest energy level has n = 1 and is called the 'ground state'. All other energy levels are called 'excited states'. The average size of the electron's orbit is also controlled by the value of n:

$$r_{\text{average}} = (0.529 \times 10^{-10}\ \text{m}) \frac{n^2}{Z}$$



Critical thinking questions

1. The hydrogen atom has atomic number Z = 1. Using the equations above for the energy and average radius of electron's orbit, complete the table below for hydrogen.

n	E_n (J)	r_{average} (m)
1	-218×10^{-20}	0.529×10^{-10}
2	-54.5×10^{-20}	2.12×10^{-10}
3	-24.2×10^{-20}	
4		
5		
6		

2. The horizontal lines on the diagram on the left show the energy levels for n = 1, 2 and 3. Using the values you calculated for E_n, add the energy levels for n = 4 – 6.

3. Describe *in words* to your neighbour what happens to the energy levels and the average size of the orbit as n increases. When you have agreed on this, write down your description in a grammatically correct sentence below.

4. What do you predict happens to the energy and orbit of the electron when n becomes very large?

Model 3: Atomic Spectroscopy

The electron in a H atom wandering around in space will be in the $n = 1$ level (the "ground state"). However, if a high voltage is passed through H_2 molecules, an excited H atom is formed in which the electron is in a level with $n > 1$. The electron in this "excited" atom quickly moves ("relaxes") to a *lower* level and the excess energy is lost ("emitted") as radiation.

For example, if the excited atom is formed with its electron in the $n = 3$ level, the electron can fall into the $n = 2$ or into the $n = 1$ level (indicated by the dotted line on the diagram on page 2). The energy lost as radiation is the *difference* between the two energy levels involved: it is equal to the *length* of the dotted line.

Critical thinking questions

1. If the excited atom is formed with the electron in the $n = 4$ level, add dotted lines to the diagram on page 2 showing how the electron can relax.

2. For each of these jumps, work out the energy of the emitted radiation in eV.

3. Calculate the electronic transitions responsible for absorbing the following energies:

   (a) Absorption wavelength: 430 nm, hydrogen atom, (one energy level involved is $n = 2$)

   (b) Absorption wavelength: 250 nm, He^+ cation, (one energy level involved is $n = 7$)

Workshop: Unit conversions for electromagnetic radiation (photons)

Radiation emitted from atoms or molecules as they 'relax' can be quantified. In the above examples you have been working with joules, the SI unit of energy. To convert between joules, wavelength and frequency, you need to use Planck's hypothesis:

$$E = h\nu = \frac{hc}{\lambda}$$

1. Label on the above equation what quantity each term represents and SI units of each term.

2. Convert the following wavelengths into frequencies (Hz), using scientific notation:

   (a) 4.33 nm

   (b) 2.35×10^{-10} m

   (c) 4.57 µm

3. Convert the following frequencies, into wavelengths in the units indicated:

(a) 4.77 GHz (m)

(b) 28.0 kHz (cm)

(c) 60. Hz (mm)

4. Calculate the energy in joules per photon and in kilojoules per mole of the following:

(a) X-rays with wavelength 25.5 nm

(b) microwaves with frequency 2.5437×10^{10} Hz

5. What are the wavelength and frequency of photons with the following energies?

(a) 745 kJ/mol

(b) 3.55×10^{-19} J photon^{-1}

Workshop: Unit conversion for wave-particles with rest mass

For wave-particles with rest mass – electrons, protons, neutrons, for example – the de Broglie relation enables conversion of the energy and wavelengths of those particle-waves to their velocity (v) and mass (m):

$$E_{kinetic} = \frac{1}{2}mv^2 \qquad \qquad \lambda = \frac{h}{mv}$$

1. Label on the above equation what quantity each term represents and SI units of each term.

2. Determine the wavelength of electrons with the following kinetic energies:

(a) 1.15×10^{-19} J

(b) 3.55 kJ/mol

(c) 7.45×10^{-3} J mol^{-1}

3. Determine the kinetic energies in joules of electrons with the following wavelengths:

(a) 3.75 nm

(b) 4.66 m

4. Calculate the kinetic energy of a neutron with a wavelength of 75 pm.