CHEM1902/4 Answers to Problem Sheet 9

1. (a) The titration is a 1:1 reaction between a weak acid and a strong base.

	CH ₃ COOH	+	H_3O^+	CH ₃ COO ⁻
initial	0.100		0	0
change	-x		+x	+x
final	0.100 – x		x	x

(i) As acetic acid is a weak acid, $[H_3O^+]$ must be calculated:

The equilibrium constant K_a is given by:

$$K_{\rm a} = \frac{[{\rm H}_3{\rm O}^+({\rm aq})][{\rm CH}_3{\rm COO}^-]}{[{\rm CH}_3{\rm COOH}]} = \frac{x^2}{(0.100 - x)}$$

As $pK_a = 4.76 = -\log_{10}K_a$, $K_a = 10^{-4.76}$. As K_a is very small, $0.100 - x \sim 0.100$ and hence:

$$x^2 = 0.1 \times 10^{-4.76}$$
 or $x = 0.00132$ M = [H₃O⁺]

Hence, the pH is given by:

$$pH = -log_{10}[H_3O^+] = -log_{10}[0.00132] = 2.88$$

(ii) Addition of the strong base OH (aq) leads to the neutralization reaction:

 $CH_3COOH(aq) + OH^-(aq) \rightarrow CH_3COO^-(aq) + H_2O(l)$

The number of moles of CH_3COO^- is equal to the number of moles of OH^- (aq) which have been added. The number of moles of CH_3COOH remaining is equal to the initial number of moles of CH_3COOH minus the number of moles of OH^- (aq) which have been added.

Initially, there is 50.0 mL of 0.100 M CH₃COOH so

 $n(CH_3COOH)_{initial} = concentration \times volume$ = 0.100 M × 0.0500 L = 0.00500 mol

25.0 mL of 0.100 M NaOH contains

$$n(OH^{-})_{added} = concentration \times volume$$

= 0.100 M × 0.0250 L = 0.00250 mol

Thus, after addition and the neutralization reaction, there are:

 $n(CH_3COOH) = n(CH_3COOH)_{initial} - n(OH)_{added}$ = (0.00500 - 0.00250) mol = 0.00250 mol

$$n(CH_3COO^-) = n(OH^-)_{added} = 0.00250 \text{ mol}$$

These quantities are now in a solution with total volume (50.0 + 25.0) mL = 75.0 mL, so their concentrations are:

$$c(CH_3COOH) = n / V = 0.00250 \text{ mol} / 0.0750 \text{ L} = 0.0333 \text{ M}$$

 $c(CH_3COO^-) = n / V = 0.00250 \text{ mol} / 0.0750 \text{ L} = 0.0333 \text{ M}$

Using the Henderson-Hasselbalch equation gives

$$pH = pK_a + \log_{10} \left(\frac{[base]}{[acid]} \right)$$
$$= 4.76 + \log_{10} \left(\frac{0.0333 \text{ M}}{0.0333 \text{ M}} \right) = 4.76$$

At the $\frac{1}{2}$ equivalence point, when the number of moles of base added is equal to $\frac{1}{2}$ the number of moles of acid originally present, the pH is equal to the p K_a .

(iii) 45.0 mL of 0.100 M NaOH corresponds to

$$n(OH^{-})_{added} = concentration \times volume$$

= 0.100 M × 0.0450 L = 0.00450 mol

Thus, after addition and the neutralization reaction, there are:

 $n(CH_3COOH) = n(CH_3COOH)_{initial} - n(OH)_{added}$ = (0.00500 - 0.00450) mol = 0.00050 mol

 $n(CH_3COO^-) = n(OH^-)_{added} = 0.00450 \text{ mol}$

These quantities are now in a solution with total volume (50.0 + 45.0) mL = 95.0 mL, so their concentrations are:

c(CH₃COOH) = *n* / *V* = 0.00050 mol / 0.0950 L = 0.00526 M

 $c(CH_3COO^2) = n / V = 0.00450 \text{ mol} / 0.0950 \text{ L} = 0.0474 \text{ M}$

Using the Henderson-Hasselbalch equation gives

$$pH = 4.76 + \log_{10} \left(\frac{0.0474 \,\mathrm{M}}{0.00526 \,\mathrm{M}} \right) = 5.71$$

(iv) The addition of 50.0 mL of 0.100 M NaOH corresponds to the equivalence point as the number of moles of added base is equal to the number of moles of acid initially present. However, the pH is *not* equal to 7 for this point of the titration of a weak acid with a strong base. 50.0 mL of 0.100 M NaOH corresponds 0.00500 mol so whilst this leads to $n(CH_3COOH) = 0.00$ mol, it also leads to $n(CH_3COO^-) = 0.00500$ mol. A solution of a weak base has pH > 7.

As the total volume is (50.0 + 50.0) mL = 100.0 mL, the concentration of CH₃COO⁻ is:

$$c(CH_3COO^{-}) = n / V = 0.00500 \text{ mol} / 0.1000 \text{ L} = 0.05000 \text{ M}$$

The calculation of the pH now follows that in Q1(b) with the concentration of 0.05000 M:

$$K_{\rm b} = \frac{[\rm OH^{-}][\rm CH_{3}COOH]}{[\rm CH_{3}COO^{-}]} = \frac{x^{2}}{0.05000 - x} = 9.24$$

As $pK_b = 9.24$, $K_b = 10^{-9.24}$. Again, K_b is very small, $0.05000 - x \sim 0.05000$ and hence:

$$x^2 = 0.0500 \times 10^{-9.24}$$
 or $x = 5.36 \times 10^{-6}$ M = [OH-]

Hence, the pOH is given by:

$$pOH = -log_{10}[OH^{-}] = -log_{10}[5.36 \times 10^{-6}] = 5.27$$

Finally, pH + pOH = 14 so

pH = 14.0 - 5.27 = 8.73

(v) Beyond the equivalence point, excess base is being added to a solution of a weak base and the pH is controlled by the excess amount of OH⁻(aq). The calculation of the pH is then *exactly* the same as in Q3a (vi) from Problem Sheet 7.

55.0 mL of 0.100 M NaOH corresponds to 0.00550 mol. Of this, 0.00500 mol is used to react with the acid. The total volume is (0.050 + 0.055) L = 0.105 L. Hence

$$[OH^{-}(aq)] = \frac{(0.00550 \text{ mol}) - (0.00500 \text{ mol})}{(0.105 \text{ L})} = 0.00476 \text{ M}$$

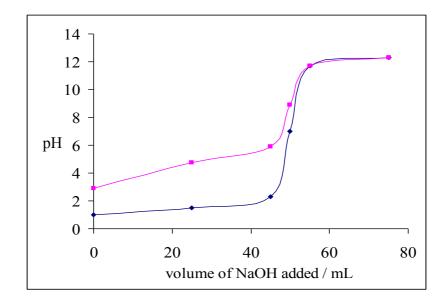
Hence, $pOH = -log_{10}(0.00476) = 2,32$ and pH = 14.00 - 2.32 = 11.7.

(vi) 75.0 mL of 0.100 M NaOH corresponds to 0.00750 mol Of this, 0.00500 mol is used to react with the acid and the total volume is (0.050 + 0.075) L = 0.125 L. Hence

$$[OH^{-}(aq)] = \frac{(0.00750 \text{ mol}) - (0.00500 \text{ mol})}{(0.125 \text{ L})} = 0.0200 \text{ M}$$

Hence,
$$pOH = -log_{10}(0.0200) = 1.70$$
 and $pH = 14.00 - 1.70 = 12.3$

(b) Using these values, the pH curve for the titration can be constructed and is shown below (in pink).



(c) The figure also includes the data from Q4 in Problem Sheet 7 for a strong acid / strong base titration (in blue).

For a weak acid / strong base titration:

- (i) The initial pH is higher. For a strong acid and a weak acid with the *same* concentration, the pH of the strong acid is lower.
- (ii) The pH at the $\frac{1}{2}$ equivalence point is equal to the p K_a of the weak acid.
- (iii) At the equivalence point, the solution contains weak base and so the pH > 7.
- (iv) After the equivalence point, the pH is determined only the concentration of excess strong base and is thus the *same* for the two titrations.
- 2. The concentration of the acid that has dissociated at equilibrium can be calculated from the pH:

	НА	+	H_3O^+	\mathbf{A}^{-}
initial	0.6		0	0
change	- <i>x</i>		+x	+x
final	0.6 - x		x	x

At equilibrium, $[H_3O^+] = [H_3O^+]_{from HA} + [H_3O^+]_{from water}$. As $[H_3O^+]_{from water} \sim 10^{-7}$, it can be assumed that the pH is due to the $[H_3O^+]_{from HA}$. Therefore:

 $pH = -log_{10}[H_3O^+] \sim -log[H_3O^+]_{from HA} = -logx = 4.0$

$$x = 10^{-4} \text{ M}$$

The initial concentration of HA is 0.6 so at equilibrium [HA] = 0.6 - $10^{-4} \sim 0.6$ M. The percentage dissociation is:

$$\frac{[\text{H}_{3}\text{O}^{+}]_{\text{from HA}}}{[\text{HA}]_{\text{equilibrium}}} = \frac{10^{-4}}{0.6} \times 100\% = \frac{10^{-4}}{0.6} \times 100\% = 0.017\%$$

3. (a) 0.2 M acetic acid

As acetic acid is a weak acid, $[H_3O^+]$ must be calculated:

	CH ₃ COOH	+	H_3O^+	CH ₃ COO ⁻
initial	0.2		0	0
change	- <i>x</i>		+x	+x
final	0.2 - x		x	x

The equilibrium constant K_a is given by:

$$K_{\rm a} = \frac{[{\rm H}_3 {\rm O}^+][{\rm CH}_3 {\rm COO}^-]}{[{\rm CH}_3 {\rm COOH}]} = \frac{x^2}{0.2 - x}$$

As $pK_a = 4.76 = -\log_{10}K_a$ so $K_a = 10^{-4.76}$. As K_a is very small, $0.2 - x \sim 0.2$ and hence:

$$x^2 = 0.2 \times 10^{-4.76}$$
 or $x = 0.0019$ M = [H₃O⁺]

Hence, the pH is given by:

 $pH = -log_{10}[H_3O^+] = -log_{10}[0.0019] = 2.7$

(b) 0.2 M sodium acetate

As CH₃COO⁻ is a weak base, [OH⁻] must be calculated in a similar way:

	CH ₃ COO ⁻	H ₂ O	1	OH⁻	CH ₃ COOH
initial	0.2	large		0	0
change	-x	negligible		+x	+x
final	0.2 - x	large		x	x

The equilibrium constant K_b is given by:

$$K_{\rm b} = \frac{[\rm OH^-][\rm CH_3COOH]}{[\rm CH_3COO^-]} = \frac{x^2}{0.2 - x}$$

SO

For an acid and its conjugate base:

$$pK_{a} + pK_{b} = 14.00$$

SO

$$pK_b = 14.00 - 4.76 = 9.24$$

As $pK_a = 4.76$, $K_b = 10^{-9.24}$. Again, K_b is very small, $0.2 - x \sim 0.2$ and hence:

 $x^2 = 0.2 \times 10^{-9.24}$ or x = 0.00001 M = [OH-]

Hence, the pOH is given by:

 $pOH = -log_{10}[OH^{-}] = -log_{10}[0.00001] = 5.0$

Finally, pH + pOH = 14.0 so

$$pH = 14.0 - 5.0 = 9.0$$

(c) A buffer that is 0.2 M in acetic acid and 0.2 M in sodium acetate

This solution contains an acid and its conjugate base so the Henderson-Hasselbalch equation can be used:

$$\mathbf{pH} = \mathbf{pK}_{\mathbf{a}} + \log_{10}\left(\frac{[\mathbf{base}]}{[\mathbf{acid}]}\right)$$

As [acetic acid] = [sodium acetate], $\log_{10}\left(\frac{[base]}{[acid]}\right) = \log_{10}(1) = 0$ and so

$$pH = pK_a = 4.76$$

4. The Henderson-Hasselbalch equation can be used for this buffer:

$$\mathbf{pH} = \mathbf{p}K_{\mathrm{a}} + \log_{10}\left(\frac{[\mathrm{base}]}{[\mathrm{acid}]}\right)$$

where [base] = [KNO₂] and [acid] = [HNO₂]. If K_a for HNO₂ = 4.00 × 10⁻⁴ M,

$$pK_a = -\log_{10}(4.00 \times 10^{-4}) = 3.40$$

Hence,

$$\log_{10}\left(\frac{[\text{base}]}{[\text{acid}]}\right) = 3.00 - 3.40 = -0.40$$

or

$$\left(\frac{[\text{base}]}{[\text{acid}]}\right) = 10^{-0.40} = 0.40$$

As the concentrations of the acid and base are the same, the volume of base must be $0.40 \times$ volume of acid. The volumes must add to give 1L:

0.71 L of HNO₂ and 0.29 L of KNO₂.

5. (a) Na₂O

 $Na_2O(s) + H_2O(l) \rightarrow 2Na^+(aq) + 2OH^-(aq) - basic$

(b) Cl_2O_7

 $Cl_2O_7(l) + 3H_2O(l) \rightarrow 2ClO_4(aq) + 2H_3O^+(aq) - acidic$