**W4 NUMBERS AND UNITS**

**SI Base Units**

In almost all cases, reporting numerical results requires that the units are specified. If sports reporters announce a new world record of “9.79 s for the 100 m sprint”, we know exactly what they are talking about. Similarly, a TV cook specifies that 25 g of butter is needed for a recipe – merely stating that “25 of butter is required” can lead to obvious ambiguity.

The preferred units in science are the **SI base units**, shown in Table W4-1 below. Other units are derived from these seven units.

**Table W4-1 SI Base Units**

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Name of Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Temperature</td>
<td>kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>mole</td>
<td>mol</td>
</tr>
<tr>
<td>Electric current</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>candela</td>
<td>cd</td>
</tr>
</tbody>
</table>

**Derived and Compound Units**

Units for other quantities are derived from these seven units. To do this, take the equation used to define the quantity and substitute the appropriate SI base unit.

**Example**

The volume \( V \) of cube is given by the length \( l \) of a side cubed:

\[
V = l^3
\]

As the SI unit of length is the metre, the SI unit of volume is the cubic metre or m\(^3\).

An extremely important derived unit is that for energy. There are many equations that relate quantities to energy changes. (eg Einstein's famous \( E = mc^2 \)) In every case, the SI unit for energy is the joule (symbol J = kg m\(^2\) s\(^{-2}\)).

If there is not a common derived unit, the unit may be made of two units.

---

* The International System of Units (abbreviated SI from the French Le Système International d’Unités)
Example

We are often interested in the energy released by a known amount of a chemical. A commonly used compound unit in chemistry is for the energy (J) released per mole (mol) of a substance. This is given the unit “joules per mole” or J mol⁻¹.

In chemistry, the base and derived units are not always convenient because we often deal with very large numbers†, such as the number of atoms in a beaker of solution, and very small numbers†, such as the size of an atom. Prefixes are added to indicate decimal fractions or multiples (Table W4-2).

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>giga</td>
<td>G</td>
<td>10⁹</td>
<td>1 GJ is about the energy of a mole of γ-rays</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>10⁶</td>
<td>4 MJ is about the energy contained in ¼ of a small chocolate bar</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>10³</td>
<td>2 kJ is about the energy corresponding to room temperature</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>10⁻¹</td>
<td>1 dm³ is commonly known as a litre</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>10⁻²</td>
<td>1 cJ is the about the energy of a mole of radio waves</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>10⁻³</td>
<td>1 mJ is about the energy used every second by a watch</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>10⁻⁶</td>
<td>1 μJ is about the energy need to light up a single phosphor on a TV screen</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>10⁻⁹</td>
<td>1 nJ is about the energy used each time a fly beats a wing</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>10⁻¹²</td>
<td>1 pJ is about the energy used in a single nerve impulse</td>
</tr>
</tbody>
</table>

Examples

(a) The length of a C–H bond is typically around 1.09 × 10⁻¹⁰ m.

\[
\text{length} = 1.09 \times 10^{-10} \text{ m} = 0.109 \text{ nm} = 109 \text{ pm}
\]

(b) Burning a mole of sucrose releases 5644 kJ of energy

\[
\text{energy per mole} = 5644 \text{ kJ mol}^{-1}
\]

† If you are unfamiliar with scientific notation for writing numbers, see the Appendix.
Q1. C–C bonds are typically around $1.54 \times 10^{-10}$ m in length and breaking a mole of bonds requires around 350 kJ of energy. Which of the one following statements concerning the bond length is correct?

A A typical C–C bond length is 154.
B A typical C–C bond length is 154pm.
C A typical C–C bond length is 154 pm.
D A typical C–C bond length is 154nm.
E A typical C–C bond length is 154 nm.

Q2. Which one of the following statements concerning the bond strength is correct?

A The C–C bond strength is 350.
B The C–C bond strength is 350kJmol⁻¹.
C The C–C bond strength is 350kJ mol⁻¹.
D The C–C bond strength is 350 kJmol⁻¹.
E The C–C bond strength is 350 kJ mol⁻¹.

**Dimensional Analysis**

You will encounter a large number of equations in your lecture notes and in textbooks. Do you need to know them all? The answer is that you do not have to. One way of reducing the number of equations is to use ‘dimensional analysis’. This is really just the reverse of the process of working out derived units used above. It enables the equation to be derived from the units. If we know the units, it helps us remember the correct form of the equation.

The two equations that you will use most often this year relate:

- mass, molar mass and moles, and
- concentration, volume and moles.

They are the equations that are most commonly written and used incorrectly by all chemists, including first years, demonstrators and lecturers.

**Mass (m), molar mass (M) and the number of moles (n):**
The unit of molar mass are “g mol⁻¹”. This is read “grams per mole”, it means grams divided by moles. Hence:

$$\text{molar mass units} = \text{g mol}^{-1} = \frac{\text{grams}}{\text{moles}}$$

From the units, we have worked out that the equation must be:

$$\text{molar mass} = \frac{\text{mass}}{\text{number of moles}}$$

$$M = \frac{m}{n}$$

The molar mass is the mass divided by the number of moles.
We can use the same trick to make sure we get the right equation when we want to work out the mass from the molar mass and the number of moles. There are three ways in which $M$ and $n$ can be combined:

$M \times n$, $\frac{M}{n}$ and $\frac{n}{M}$ and it can be easy to pick the wrong ones.

A quick dimensional analysis check helps:

- $M \times n$ has units of $g \ mol^{-1} \times \text{mol} = g$
- $\frac{M}{n}$ has units of $\frac{g \ mol^{-1}}{\text{mol}} = g \ mol^{-2}$
- $\frac{n}{M}$ has units of $\frac{\text{mol}}{g \ mol^{-1}} = g^{-1} \ mol^{2}$

We are trying to calculate a mass (unit = “g”). This means that our formula must be:

$$m = M \times n$$

**The mass is the molar mass times the number of moles.**

A convenient way to help remember these relationships is to use the ‘triangle method’:

Use a finger to cover up the quantity you want to calculate. The other two values show you how to do the calculation.

To calculate the mass ($m$), cover it up and you are left with “$n \ M$”, hence $m = n \times M$.

To calculate the molar mass ($M$), cover it up and you are left with “$m \ n$”, hence $M = \frac{m}{n}$.

**Concentration ($c$), volume ($V$) and the number of moles ($n$)**

The units of concentration are “mol L$^{-1}$”. This is read as “moles per litre” and means moles divided by litres. Hence:

$$\text{concentration units} = \text{mol L}^{-1} = \frac{\text{moles}}{\text{litres}}$$

From the units, we have worked out that the equation must be:

$$\text{concentration} = \frac{\text{number of moles}}{\text{volume}}$$

$$c = \frac{n}{V}$$

**The concentration is the number of moles divided by the volume.**
Again, we can use the same trick to make sure we get the right equation to calculate the number of moles from the concentration and the volume. The three possible combinations of $c$ and $V$ and their units are:

- $c \times V$ has units of $\text{mol L}^{-1} \times \text{L} = \text{mol}$
- $\frac{c}{V}$ has units of $\frac{\text{mol L}^{-1}}{\text{L}} = \text{mol L}^{-2}$
- $\frac{V}{c}$ has units of $\frac{\text{L}}{\text{mol L}^{-1}} = \text{mol}^{-1} \text{L}^{-2}$

We were trying to calculate the number of moles (unit = “mol”). Therefore, the first is correct and the formula required is

$$n = c \times V$$

The number of moles is the concentration times the volume.

Q3. Complete the triangle below showing the relationship between $n$, $c$ and $V$.

![Triangle Diagram]

Q4. Use a dimensional analysis to determine the equation used for calculating the number of moles from the mass and the molar mass.

$$n = \frac{m}{M}$$

Q5. Use a dimensional analysis to determine the equation used for calculating the volume from the concentration and the number of moles?

$$V = \frac{c}{n}$$

Q6. Check your answers using the triangles.

(\text{\em Even if you already know the answers, use a dimensional analysis to show the units are correct.})
Chemists and SI Units

The SI unit of volume is the cubic metre. This may be a useful unit for people filling up hot-air balloons or swimming pools; it is much less handy for chemists and biologists. We tend to use the litre (symbol L) when measuring and calculating for no better reason than this means we are working with simpler numbers.

For similar reasons, we use the gram (symbol g) for mass instead of the kilogram, the kilojoule (symbol kJ) for energy instead of the joule and the kilopascal (kPa) or atmosphere (atm) for pressure instead of the pascal.

Q7. How many litres are there in one cubic metre? (Hint: 1 m = 1000 cm and 1 L = 1000 cm³).
Appendix

Very large and very small numbers are often encountered in Chemistry. Writing them out in full can be very time consuming and can lead to text which is difficult to read. Instead, scientific notation (also called exponential notation) is used. Numbers are expressed in the form:

\[ N \times 10^n \]

where \( N \) is a number between 1 and 9.9999… and \( n \) is the exponent.

**Large numbers - positive exponents**

Written long hand, the Avogadro constant (to 4 significant figures) is

\[ 60 220 000 000 000 000 000 000 \text{ mol}^{-1} \]

In scientific notation, it becomes:

\[ 6.022 \times 10^{23} \text{ mol}^{-1} \]

(read "six point zero two two times ten to the twenty-three per mole")

The exponent tells us how many times the number 6.022 is multiplied by 10 to give the long form.

Alternatively, the exponent can be thought of as the number of times the decimal point in the long hand form must be moved to the left in order to obtain a number between 1 and 9.9999…..

One million = 1000000.0

The decimal point must be moved 6 places to the left to give the number 1.

In scientific notation, one million is \( 1.0 \times 10^6 \)

**Small numbers - negative exponents**

Written long hand, the mass of a hydrogen atom is

\[ 0.000 000 000 000 000 000 000 000 001 66 \text{ kg} \]

In scientific notation, it becomes

\[ 1.66 \times 10^{-27} \text{ kg} \]

(read "one point six six times ten to the minus twenty-seven kilograms")

The negative exponent tells us how many times the number 1.66 is divided by 10 to give the long form.

Alternatively, the exponent can be thought of as the number of times the decimal point in the long hand form must be moved to the right in order to obtain a number between 1 and 9.9999…..

One millionth = 0.000001

The decimal point must be moved 6 places to the right to give the number 1.

In scientific notation, one millionth is \( 1.0 \times 10^{-6} \).
Scientific notation on calculators
Scientific calculators have a key labelled 'EXP' (occasionally 'EE') for entering numbers in scientific notation.

To enter the number "$6.022 \times 10^{23}$", the following sequence is pressed:

(i) Type “6.022”
(ii) Press the $\text{EXP}$ button
(iii) Type “23”

To enter the number "$1.66 \times 10^{-27}$", the following sequence is pressed with the '+/-' button used to change the sign of the exponent to negative:

(i) Type “1.66”
(ii) Press the $\text{EXP}$ button
(iii) Press the $\text{+/-}$ button
(iv) Type “27”

On some calculators, steps (iii) and (iv) may be done in either order.