W5 PRECISION IN CALCULATIONS

Uncertainty and Significant Figures

We have imperfect senses and use imperfect measuring devices. All measurements we make include some *uncertainty*. When we use these data, it is important that we recognise what that uncertainty is and do not exaggerate its precision.

For example, if a salesperson tells you a car is 11 years old, it would be unwise to calculate its true age as 11 × 365 = 4015 days and even more foolish to work out its age in minutes, just because the calculator allows you to do so. More likely, the seller means that the car is more than 10 ½ years old and less than 11½ years old. The true age is 11 ± 0.5 years, where the '±' symbol expresses the uncertainty. The real age may be *anywhere* within this range.

Generally, when reporting data, the '±' symbol is not given. Rather, an uncertainty of *one unit in the right most digit* is assumed. The number of digits that are reported or recorded, both the certain and uncertain ones, are called **significant figures** (often abbreviated to **sf, s.f.**, **sig figs** or **sig. figs.**).

The statement, "the car is 11 years old" means that the age is known to a precision of two significant figures, no more and no less. If the statement is made that the car is 11.5 years old, we can assume the age is known to greater precision. When using the reported age to value the car, we must remember the uncertainty.

When you take measurements or use them in calculations, you must know the number of digits that are significant.

> **The number of significant figures that data is known to and the number of significant figures you should use are *not* given explicitly in this lab manual, in most university level textbooks, in lectures notes and *in exams*.**

To determine the number of significant figures in a measurement, follow the procedure below.

> **All digits are significant, except for zeros that are not measured but are used only to position the decimal point.**
>
> 1. **Make sure that the measurement has a decimal point. (Add one if necessary.)**
> 2. **Start at the left of the number and move right until you reach the first non-zero digit.**
> 3. **Count that digit and every one to its right as significant.**
> 4. **Zeros that *end* a number and lie *after* or *before* the decimal point are significant.**

Example

The significant figures in the numbers below are underlined:

(i) 0.00$\underline{62}$ L (2 sf) (ii) $\underline{57.6}$ s (3 sf) (iii) 0.0$\underline{1033}$ m (4 sf)

You should take care with zeros that end a number (rule 4):

(iv) $\underline{1.050}$ L (4 sf) (v) 0.0$\underline{6000}$ s (4sf) (vi) $\underline{100.0}$ m (4 sf) (vii) $\underline{500}$. kg (3sf)

Example (vii) was straightforward as a decimal point was included in the number, and this is the modern convention. Hence, $\underline{5}$00 kg has only one significant figure. The situation is even clearer if scientific notation is used. Thus;

(viii) $\underline{5.00} \times 10^2$ kg (3 sf) (ix) $\underline{5.0} \times 10^2$ kg (2 sf) (x) $\underline{5} \times 10^2$ kg (1 sf)

Although the use of a terminal decimal point (as in (vii) above) is the convention, you should take care as many books, websites and lecturers do not always stick to it.

Q1. Complete the table below by underlining the numbers which are significant and writing the number of significant figures for each value.

Value	**Number of significant figures**
32.010 g	
0.0100 L	
1300.	
1300	
1300.00	

Q2. The smallest atom is helium with a radius of 31.0 pm. Using scientific notation[*], write this number to (a) 3 sf, (b) 2 sf and (c) 1 sf.

Value	**Number of significant figures**
	1
	2
	3

[*] see the Appendix in W4

Using Significant Figures in Calculations

It is important not to claim more certainty in an answer than is possible given the uncertainty in the original data. The temptation when using calculators is to produce too many significant figures. If you have too many, **round off**.

> **The *least* certain measurement sets the limit on the certainty for the calculation and the number of significant figures in the answer.**
>
> **5. When multiplying or dividing, the answer should contain the *same* number of significant figures as the measurement with the *fewest* significant figures.**

Example
The concentration (c) of salt in seawater is measured to be 0.564 mol L^{-1}. The number of moles of salt (n) in a volume (V) of 1.5 L of seawater is:

$$n = c \times V = 0.564 \times 1.5 = 0.85 \text{ mol}$$

The calculator may show 0.846, but as the volume is only reported to 2 sf, the answer cannot be known more accurately than 2 sf.

> **6. When adding or subtracting, the answer should contain the *same* number of decimal places as the measurement with the *fewest* decimal places.**

Example
If the contents of two beakers, one containing 20.1 mL of water and one containing 30.02 mL of water, are added together the total volume of water is:

$$V = 20.1 + 30.02 = 50.1 \text{ mL}$$

The calculator may show 50.12, but the first beaker contains 20.1 ± 0.05 mL so that such precision in the second decimal place cannot be justified.

> **7. When rounding off, if the digit removed is:**
> **(i) *more than 5*, increase the preceding number by 1**
> **(ii) *less than 5*, do not change the preceding number**
> **(iii) *equal to 5*, increase the preceding number by 1 *if it is odd* and do not change the preceding number *if it is even*.**

Read rule 7(iii) again – it may surprise you. When rounding, numbers ending in 1, 2, 3 and 4 are always rounded down and those ending in 6, 7, 8 and 9 are always rounded up. Assuming that these cases happen with equal frequency, the average value of a large sample of measurements is unaffected. If numbers ending in 5 were always rounded down, the average would be reduced! If numbers ending in 5 were always rounded up, the average would be increased! Rule 7(iiii) ensures that the average is unaffected.

Example
(i) 0.49499 mL rounds to 0.49 mL (2 sf) and to 0.495 mL (3 sf).
(ii) 11.65 mL rounds to 11.6 mL (3 sf), but 11.75 mL rounds to 11.8 mL (3 sf).

> **8. In a multi-step calculation, carry forward all figures in each step and round off at the end. The memory function on your calculator is very useful!**
> **9. In a multi-step calculation where you are asked to report values at each step, round these off when reporting, but carry the full number through for the next step.**
> **10. Do not be *too* worried if your final value differs in the last decimal point from one in a book, lecture notes or exam answer. This may be caused by rounding in an earlier step.**

Q2. Perform the following calculations, giving the final answer to the appropriate number of significant figures.

(a) The heights of three students are 2 m, 1.5 m and 1.6 m. What is their average height?

average height =

(b) NaCl has a molar mass of 36.458 g mol^{-1}. How many moles are there in 25 g?

$$n = \frac{\text{mass}}{\text{molar mass}} =$$

(c) This quantity of NaCl is dissolved in 200.0 mL of water. What is the concentration?

$$c = \frac{\text{no. of moles}}{\text{volume (in litres)}} =$$

Demonstrator's Initials	