CHEM1901/3 Problem Sheet 2 (Week 2)

1. Balance the following nuclear reactions and identify the missing nuclide or nuclear particle. (A periodic table is provided in the handbook.)

   e.g. ${}^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$

   (a) ${}^{20}_{10}Ne + {}^{1}_{0}n \rightarrow {}^{20}_{9}F + ?$

   (b) ${}^{15}_{7}N + {}^{1}_{1}p \rightarrow ? + {}^{1}_{0}n$

   (c) ${}^{16}_{8}O + {}^{1}_{1}p \rightarrow {}^{13}_{7}N + ?$

2. ${}^{19}_{9}F$ is a stable nuclide. One of the following isotopes of fluorine undergoes radioactive decay by β^- emission and one decays by β^+ emission. Predict which is which and write balanced equations for the decay reactions.

   (a) ${}^{18}_{9}F$

   (b) ${}^{20}_{9}F$

3. Calculate the radiocarbon age of a sample whose ${}^{14}C$ activity is 0.344 of a modern standard.

4. Calculate the molar activity of tritium (in Curie), given its half-life of 12.26 years.
   [1 Ci = 3.70×10^{10} disintegrations per second.]

5. Arrange the following elements in order of increasing ionization energy:

   Ne, Na, C, Mg, N, F

6. Identify three elements whose atomic radii are similar to that of Li.

7. Identify the largest and smallest of all neutral atoms.

Optional.

(a) Read the following article on Einstein's $E = mc^2$ equation.

(b) Calculate (i) the change in mass, or **mass defect,** (in a.m.u.) which occurs when six H atoms and six neutrons fuse together to give one ${}^{12}C$ atom and (ii) the energy equivalent (in kJ mol^{-1}). [Masses in amu: ${}^{1}H$ 1.007825, ${}^{1}n$ 1.008665, ${}^{12}C$ 12.000000]

$E = mc^2$ for the Chemist: When Is Mass Conserved?

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In 1905 Albert Einstein derived an equation that expresses a relationship between mass and energy *(1).* These two quantities, which were previously thought to be independent of one another, became linked by $E = mc^2$. In spite of its simplicity, the equation is often misinterpreted in textbooks and the popular science literature. Authors in both physics *(2–12)* and chemistry *(13–19)* have attempted over many years to correct the misconceptions. This article is one such effort. It asks to what extent is mass conserved in the reactions of physics and chemistry. The question can be used to challenge students and develop their language and critical thinking skills.

A Brief History and Apparent Contradictions

Although $E = mc^2$ began as only a minor component of the theory of special relativity, it soon became its most widely recognized feature. As modern physics developed in the 20th century, nuclear and subatomic particle reactions were discovered that could be used to test the equation. The data they provided were often interpreted to mean that mass is simply one form of energy and that it can be converted into other forms, such as heat and work. If this were true, then mass could no longer be the conserved quantity it had always been.

Chemists viewed the famous equation from a distance and perhaps with some suspicion. The 18th century discoveries of Antoine Lavoisier provided convincing evidence that the reactants and products of a chemical reaction always have identical masses *(20).* Belief in conservation of matter enabled the atomic theory to be developed through the 19th century. Each chemical element was assigned an atomic mass that was assumed not to change as its atoms underwent chemical change. Any heat, work, or other energy produced by a reaction was said to have been derived from chemical energy. Mass did not enter into the discussion of energy. Chemists must have been tempted to conclude that $E = mc^2$ had no relevance for their discipline.

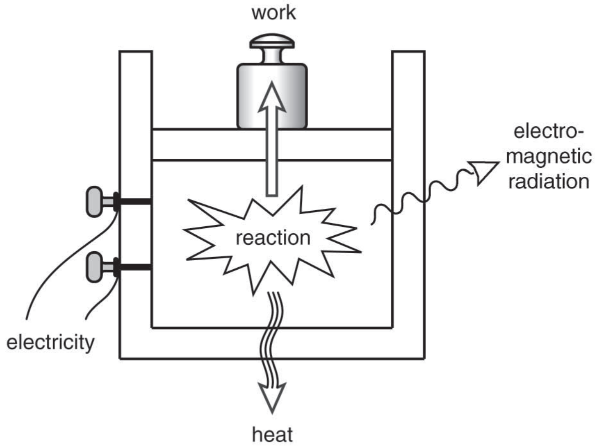

Figure 1. Various forms of energy emitted by a reaction in a closed system. An equivalent quantity of mass accompanies each release of energy.

Today's general chemistry students have good reason to be confused about mass and energy conservation. Early in the course they are told that mass is neither created nor destroyed in chemical reactions. This principle is the basis for determining compound formulas and performing stoichiometric calculations. When thermochemistry is introduced, students learn that energy, too, cannot be created or destroyed, although it can be converted from one form to another. Thus, both properties of matter are said to have their own conservation law. Near the end of the course, nuclear reactions are introduced and everything appears to change. Mass is now said to be able to be converted into energy. If this is true, then both conservation laws must be abandoned or perhaps combined in some fashion. Is it possible that nuclear reactions and chemical reactions play by a different set of rules?

Mass–Energy and Its Conservation

The theory of special relativity boldly asserts that mass and energy are not the independent quantities they were once thought to be. Rather, they are two measures of a single quantity. Since that single quantity does not have its own name, it is called *mass–energy*, and the relationship between its two measures is known as *mass–energy equivalence.* We may regard c^2 as a conversion factor that enables us to calculate one measurement from the other. Every mass has an *energy-equivalent* and every energy has a *mass-equivalent*. If a body emits energy to its surroundings it also emits a quantity of mass equivalent to that energy. The surroundings acquire both the energy and mass in the process.

Figure 1 illustrates several ways a reaction in a closed system can emit energy to its surroundings. Since the system is closed, we might ask how the surroundings are able to acquire mass along with the energy they receive. If the energy is received as *heat*, the increase in mass of the surroundings results because its atoms move faster as they warm. The mass of any body increases with its velocity according to special relativity. When the energy is received as *electricity*, the atoms in the surroundings move faster if the electrical energy produces heat or they acquire chemical energy if an electrolytic reaction occurs. In either case the atoms increase in mass. If the energy is received as *electromagnetic radiation*, the surroundings gain the mass of the emitted photons. A photon has an energy, $E = h\nu$, where ν is the frequency of the radiation. Hence, it has a mass, $m = h\nu/c^2$. We should not be surprised to learn that a photon has mass. Astronomers report that a beam of light is bent by gravity as it passes a massive body such as the sun. Recall also that light cannot escape from the gravity of a black hole. Finally, if the energy is received as *work*, the surroundings gain mass because their potential energy increases. For example, a weight placed on the lid of the system will be lifted. The two-body system consisting of the weight and the earth will acquire gravitational energy. In similar fashion, the mass of a rubber band increases a tiny degree as it is stretched. Energy in any form always has mass.

What became of the laws of conservation of mass and conservation of energy when physicists discovered that mass and energy are two measures of the same quantity? Each of the laws survived. It became clear, however, that they are only alternative expressions of a single law. The *law of conservation of mass–energy* requires that mass–energy cannot be created or destroyed. It merges the two previously independent laws into one. A common misconception is that the conservation of mass–energy only requires that the *total* of mass and energy remains constant and that the two quantities can be converted into one another. In truth, taking the sum of mass and energy of a body is an exercise in double counting.

Incidentally, we chemists are well acquainted with other quantities that can be expressed by two different measures. The quantity of water in a beaker, for example, can be measured by its mass or its volume. The density of water is the conversion factor used to calculate one measurement from the other. Note that this conversion factor only enables us to change the units used to express the quantity of water. It does not imply that we can transform mass of water into volume of water. Similarly, we cannot transform mass into energy.

We will need a value for c^2 when performing calculations. The speed of light is 2.997925×10^8 m/s in SI units. Therefore, $c^2 = 8.98755 \times 10^{16}$ m^2/s^2. If it is used as a conversion factor, c^2 should be expressed in mass and energy units. Recall that the SI unit for energy is the joule, whose definition is 1 J = 1 kg m^2/s^2. Thus, $c^2 = 8.98755 \times 10^{16}$ J/kg. Table 1 lists other values for c^2 expressed in a variety of mass and energy units.

Nuclear and Subatomic Particle Reactions

Mass–energy conservation can be experimentally tested by the energetic reactions of nuclei and subatomic particles. A frequently discussed reaction is the fission of a lithium-7 nucleus when it is bombarded by a proton:

$$^{1}\mathrm{p} + {}^{7}\mathrm{Li} \rightarrow 2\,{}^{4}\mathrm{He} + \text{kinetic energy}$$

The two alpha particles produced have kinetic energy that is quickly transferred to the surroundings. The reaction was first studied in the 1930s by Cockcroft and Walton *(21).* Table 2 gives an accounting of mass–energy for the reaction. The experimental data on the first line are the rest masses of the nuclei and the net energy transferred to the surroundings *(22).* This net energy is equal to the kinetic energy of the alpha particles minus the kinetic energy of the bombarding proton. In the second line, each of these measurements has been converted into its mass-equivalent or its energy-equivalent by use of the appropriate value of c^2 from Table 1.

Analysis of the data begins in the third line of the table. The total mass of the reactants and the total mass of the products have been calculated. The latter total includes the mass-equivalent of the energy released to the surroundings. The two totals are identical within the certainty of the measurements, hence, they demonstrate that mass is conserved in the reaction. Similarly, energy conservation is demonstrated in the fourth line, where the same totals have been calculated except each measurement is now expressed as an energy. We should expect that mass and energy are both conserved, since they are two measures of the same quantity.

The last line of the table demonstrates how the experimental data can be misinterpreted. The difference between the rest mass of the reactants and products has been calcu-

Table 1. The Mass–Energy Conversion Factor Expressed in Various Units

		Energy Unit			
		MeV	J	cal	kWh
Mass Unit	amu	931.494 MeV/amu	1.49242×10^{-10} J/amu	3.56696×10^{-11} cal/amu	4.14561×10^{-17} kWh/amu
	g	5.60959×10^{26} MeV/g	8.98755×10^{13} J/g	2.14808×10^{13} cal/g	2.49654×10^{7} kWh/g

Table 2. Mass–Energy Account for the Nuclear Fission of Lithium-7

Quantity	^{1}p	+	^{7}Li	→	2 ^{4}He	+	energy
Experimental Data	1.0073 amu		7.0144 amu		2 x 4.0015 amu		17.3 MeV
Mass-Equivalent or Energy-Equivalent	938.3 MeV		6533.9 MeV		7454.7 MeV		0.0186 amu
Demonstration of Mass Conservation	8.0217 amu				8.0216 amu		
Demonstration of Energy Conservation	7472.2 MeV				7472.0 MeV		
Misconception of Mass Converted to Energy	0.0187 amu or 17.4 MeV						17.3 MeV

lated, and the energy equivalent of that lost mass is also entered. This calculated energy agrees well with the energy released to the surroundings. We should not conclude, however, that mass is converted or transformed into energy. Recall that the energy released to the surroundings carries mass with it. Mass is conserved in the reaction; it does not become energy.

It is useful to examine the equations that apply to the mass–energy accounting just discussed. Consider the general reaction

$$\mathrm{A} + \mathrm{B} \rightarrow 2\mathrm{C} + \text{energy}$$

Conservation of mass requires that the particle masses, *M*, and the energy released to the surroundings, *E*, follow the equation

$$M_\mathrm{A} + M_\mathrm{B} = 2M_\mathrm{C} + \frac{E}{c^2}$$

If we multiply by c^2 we obtain the requirement for conservation of energy:

$$M_\mathrm{A}c^2 + M_\mathrm{B}c^2 = 2M_\mathrm{C}c^2 + E$$

This equation can be rearranged to give

$$(M_\mathrm{A} + M_\mathrm{B} - 2M_\mathrm{C})c^2 = E$$

Each of these equations is confirmed by the experimental data of Table 2. Note that the last of the equations affirms that the mass lost by the reacting system has an energy-equivalent equal to the energy gained by the surroundings. It does not imply, however, that mass has been converted into energy.

An analogy from everyday life can be used to explain why mass may appear to be converted into energy in reactions. Imagine the process illustrated in Figure 2 involving a bag full of gold coins. An experiment is performed in which the bag is weighed, two coins are removed from it, and it is weighed again. From the data shown in the figure it can be determined that each coin has a mass of 24 g. Hence, the conversion factor, 1 coin per 24 g, could be used to convert any measure of mass into the equivalent number of coins. Both mass and the number of coins are conserved in the experiment. These two measures are simply alternative ways of expressing quantities of gold. It would be incorrect to conclude that mass has been converted into coins in the experiment, although a quick glance at the data in the figure might give that impression.

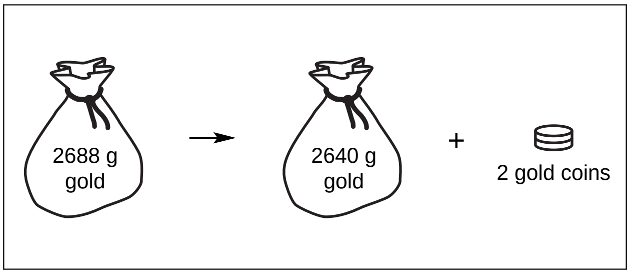

Figure 2. Experiment in which two gold coins are removed from a bag of coins. Total mass of gold and total number of coins are both conserved in the process, but the data available may suggest that mass is converted into coins.

The most energetic of all reactions are those in which matter and antimatter undergo mutual annihilation. Consider the reaction between an electron and a positron to produce two photons:

$$\mathrm{e}^- + \mathrm{e}^+ \rightarrow 2h\nu$$

Since the reacting particles are annihilated, all their mass is lost. Mass conservation is maintained nonetheless because the lost mass is carried by the photons, and it is finally acquired by the surroundings when the photons are absorbed. If we choose to define *matter* in such a way that it includes protons and electrons but not photons, then we can say that matter has been destroyed in this reaction. Mass, however, is not destroyed.

Chemical Reactions

Chemical reactions are much less energetic than the reactions discussed above. Nevertheless, they are not exempt from the requirement of mass–energy conservation. When chemical reactants undergo a reaction that emits energy, they must lose an equivalent quantity of mass. Let us illustrate the process with a reaction well known for its release of energy, the explosion of nitroglycerine. The mass lost by the reactants can be calculated from chemical thermodynamics most easily if we imagine the reaction to begin and end with all substances in their standard states and at 25 °C:

$$\begin{aligned}&4\mathrm{C_3H_5N_3O_9(l)} \rightarrow \\ &\qquad 6\mathrm{N_2(g)} + 12\mathrm{CO(g)} + 10\mathrm{H_2O(l)} + 7\mathrm{O_2(g)}\end{aligned}$$

Using standard enthalpies of formation (*23*) we obtain $\Delta H = -2700$ kJ. Since the reaction occurs at constant pressure, the relationship $\Delta E = \Delta H - P\Delta V$ applies. This equation states that the energy lost by the reacting system is the sum of the heat it emits and the work it does. The explosive power of nitroglycerine is largely due to the work done on the surroundings by the rapid expansion of the hot gases formed. The approximation $P\Delta V = \Delta nRT$, where Δn is the change in the chemical amount (number of moles) of gas, gives $P\Delta V =$ 62 kJ. Therefore, $\Delta E = -2762$ kJ. Applying the appropriate mass–energy conversion factor we find that the energy lost is equivalent to 3.074×10^{-8} g. In summary, the products of the reaction have 2762 kJ less energy and 3.074×10^{-8} g less mass than the reactants.

The loss of mass that accompanies the reaction just discussed is too small to be detected by even the best measuring instruments, and the same is true for all chemical reactions. This explains why chemists in the laboratory have never found an exception to the principle handed down by Lavoisier that no change of mass occurs in the reacting matter. Strictly speaking, however, the mass of the matter in the reacting system is not exactly constant. In recognition of this fact we should say that no *detectable* change of mass occurs in the reacting matter.

The gold coin analogy discussed earlier can be changed in such a way as to explain why the mass of reacting matter in chemical reactions appears not to change. Figure 3 illustrates a process in which two gold coins are now removed from a truck containing a very large quantity of coins. The truck mass is so large that it appears to be unchanged in the process. The fact that each coin has a mass of 24 g is still true, but it cannot be deduced from the available data. The mass of gold in the truck is not exactly conserved, but the data available give that impression.

Mass Defect of Nuclei and Molecules

General chemistry textbooks commonly introduce $E = mc^2$ when discussing the stability of atomic nuclei. All nuclei have less mass than the protons and neutrons of which they are composed. The *mass defect* of a nucleus is the mass lost when it is formed from these nucleons. The simplest example would be the formation of hydrogen-2:

$$^{1}\text{p} + {}^{1}\text{n} \rightarrow {}^{2}\text{H} + \text{energy}$$

The rest masses of a proton and neutron are 1.007276 amu and 1.008665 amu, respectively, and that of an ^{2}H nucleus is 2.013553 amu *(23).* Hence, the mass defect is 0.002388 amu. This mass has an energy-equivalent of 3.564×10^{-13} J. This is the energy released by the reaction. It is called the *binding energy* of the nucleus because it would be required to break the bond between the two nucleons.

We can extend the concept of mass defect and binding energy to chemistry. Consider the bonding of atoms required to form a methane molecule:

$$\text{C} + 4\text{H} \rightarrow \text{CH}_4 + \text{energy}$$

The energy released by this reaction is 2.916×10^{-18} J/molecule *(24).* This *chemical* binding energy has a mass-equivalent of 1.954×10^{-8} amu. Thus, a methane molecule has a *chemical* mass defect of 1.954×10^{-8} amu. Its mass is that much less than of the atoms of which it is composed. Figure 4 illustrates the mass defect of the ^{2}H nucleus and the CH_4 molecule. The ^{2}H nucleus has 0.12% less mass than its nucleons, whereas the CH_4 molecule has only 0.000031% less mass than its atoms.

The carbon-12 atom is special in that its mass is set by definition. The rest mass of ^{12}C is exactly 12 amu. We can calculate the mass of the atom under other conditions, such as when it has gained kinetic energy by being warmed or when it has lost chemical energy owing to bonding to other carbon atoms to produce graphite or diamond. Such calculations have been previously reported *(16)* and the results are summarized in Table 3. Graphite and diamond can be said to have a mass defect. Although the defect is insignificant for practical purposes, we chemists should occasionally remind ourselves that the mass of an atom depends upon its chemical state.

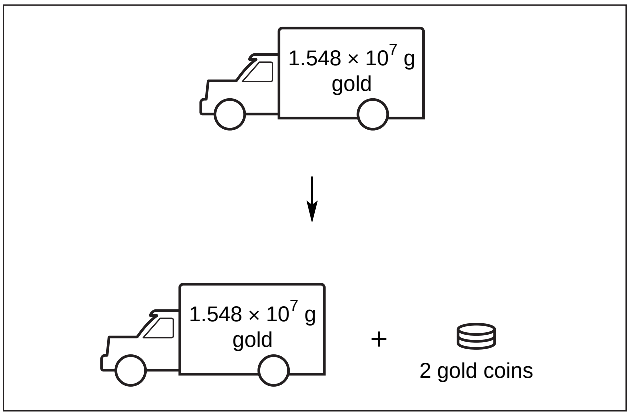

Figure 3. Experiment in which two gold coins are removed from a truck full of coins. The mass of gold in the truck appears to be constant because it decreases by a quantity too small to be detected.

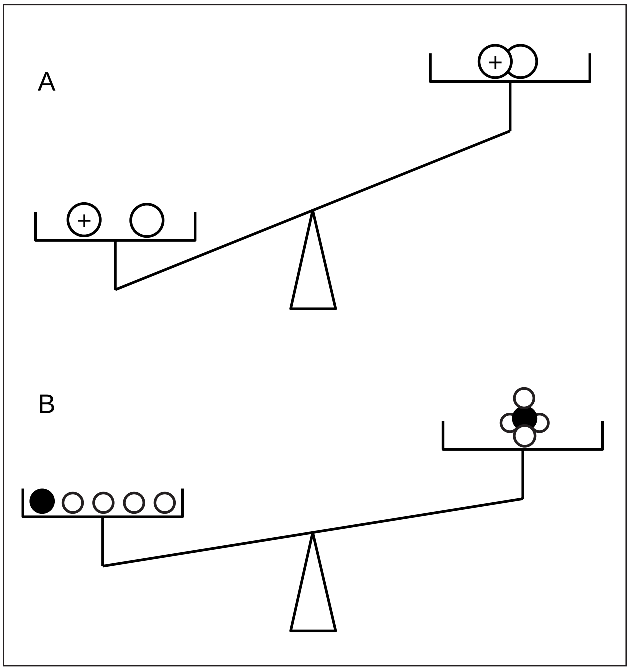

Figure 4. The mass defect of (A) an ^{2}H nucleus and (B) a CH_4 molecule demonstrated by a hypothetical experiment using a double-pan balance.

Table 3. Atomic Mass of Carbon-12 in Various Chemical States

Physical State	Atomic Mass/amu
Isolated atoms at rest	12.000000000000
Isolated atoms at 298 K	12.000000000045
Graphite at 298 K	11.999999992116
Diamond at 298 K	11.999999992137

Fraction of Mass Lost

Any reaction that emits energy to its surroundings can be characterized by the fraction of mass lost by the reactants in the process. If we use the definition $\Delta m = m_{\text{products}} - m_{\text{reactants}}$, then the mass lost is $-\Delta m$, and the fraction of mass lost is $-\Delta m/m_{\text{reactants}}$. Figure 5 displays the fraction of mass lost for various reactions on an exponential plot. The figure includes several reactions we have discussed. Also included are ^{2}H fusion, ^{235}U fission, hydrogen gas combustion, and the freezing of water.

The figure shows that reactions differ greatly in their fraction of mass lost and, hence, in the quantity of energy they emit. At the top of the figure is the highly energetic annihilation reaction followed by three nuclear reactions. At the bottom are three chemical reactions and a change of physical state. As we have discussed, the mass lost by these less-energetic reactions must be calculated from the energy lost; it cannot be measured directly.

Equations Speak for Themselves

Let us pause to ask what any mathematical equation means when it states that two quantities are equal. It does not imply that one quantity can be converted into the other. Consider, for example, the equation from geometry,

$$C = \pi d$$

which gives the relationship between the circumference, *C*, and diameter, *d*, of a circle. Either of these quantities can serve as a measure of the size of the circle. The equation affirms that if either one increases the other must also increase. If either is known the other can be calculated from it, and π serves as a conversion factor for the calculation. Most importantly, we know that diameter never disappears to be replaced by circumference. The equation does not say that diameter can be transformed into circumference. Similarly, the equation $E = mc^2$ does not say that mass can be converted into energy.

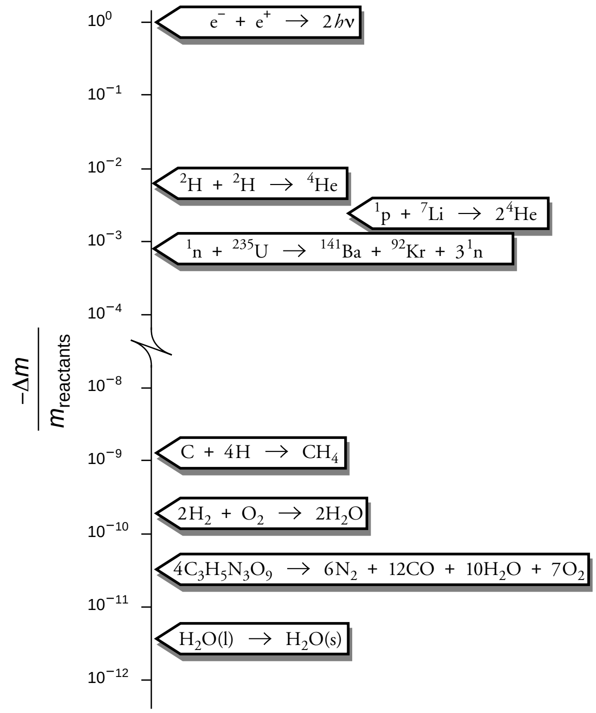

Figure 5. Fraction of mass lost by the reactants in various reactions that emit energy.

A different style of equation is needed in situations where one quantity can be converted into another. For example, when a ball is thrown straight up into the air its kinetic energy, KE, is changed into potential energy, PE, as it rises. The equation that expresses that fact is

$$\text{KE} + \text{PE} = \text{constant}$$

It is clearly of a different form. Recall that the equation we are discussing is $E = mc^2$; it is not $E + mc^2$ = constant. In summary, equations will speak for themselves if we are willing to listen.

Laws and Principles

We can now formulate a universal set of natural laws and also a principle useful in the practice of chemistry. The first and most comprehensive law is the

> *Law of Mass–Energy Conservation: Mass–energy cannot be created or destroyed.*

This law has no known exceptions. Since mass–energy can be measured by either mass or energy, it can be made more specific through two corollaries:

> *Corollary A. Law of Conservation of Mass: Mass cannot be created or destroyed.*
>
> *Corollary B. Law of Conservation of Energy: Energy cannot be created or destroyed.*

Both the reacting system and its surroundings must be considered when applying these three laws. They pertain to all the reactions of both physics and chemistry. A special situation arises when the requirement of conservation of mass is applied to chemical reactions:

> *Principle of Constant Mass: Chemical reactions occur with no detectable change in the mass of the reacting matter.*

This principle applies to the reacting system only. It is the modern update of Lavoisier's principle of conservation of matter. Its usefulness to the practice of chemistry is undeniable. It should not be classified as a law, however, because it only speaks about what is detectable. It does not express a universal truth.

Summary and Suggestions

The question asked in the title of this article has a simple answer: Mass is always conserved if we take into account both the reacting system and its surroundings. Mass is lost by the reacting system in any process that emits energy, but an equal quantity is acquired by the surroundings. Energy is conserved as surely as mass is. In fact, mass and energy are merely alternative measures of the single quantity known as mass–energy. The equation $E = mc^2$ enables us to calculate the energy-equivalent of any mass or the mass-equivalent of any energy.

Discussions of nuclear and subatomic particle reactions often lead to the misconception that mass has been converted into energy. In truth, a full accounting of both the reacting system and its surroundings will show that both mass and energy have been conserved. At best, the claim that mass can be converted into energy is a simplification that focuses attention on the most recognizable features of a reaction. If taken literally, however, it is incorrect.

In chemical reactions the mass lost by the reacting system is always too small to be detectable. This is true even for reactions that emit an easily measured quantity of energy to their surroundings. We should not conclude, however, that these reactions are exempt from mass–energy equivalence. Of course, the fact that the mass of the reacting matter is constant for all practical purposes greatly simplifies the practice of chemistry.

At what point should these concepts be introduced into the traditional general chemistry course? When atomic mass, the elemental composition of compounds and reaction stoichiometry are introduced, students need to understand that for all practical purposes the mass of an atom does not change with its chemical environment. At this time, the principle of constant mass should be introduced. Later in the course when the subject of thermodynamics enters, students need to know that energy is conserved when it is transformed from one form to another. The law of conservation of energy makes this clear. Finally, when nuclear binding energies and nuclear reaction energies are discussed late in the course, the subject of mass–energy equivalence should be presented along with the law of mass–energy conservation and its corollaries. The beauty of $E = mc^2$ can then be appreciated.

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